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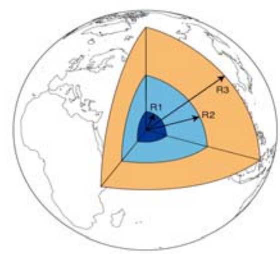
Rapidly Rotating Magneto-Convection:
Heat-flux, vortex size and velocity scaling



UK-MHD - Nice, april 2004

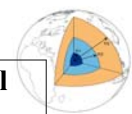
Frame

1 – <u>The non-magnetic case</u>
- Our Quasi-Geostrophic (QG) numerical code
- The onset of rotating convection
- Considerations about the vortex size
- Scaling of the convective motions

2 – <u>Introduction of an azimuthal magnetic field</u>
- Our Hybrid QG / 3D code
- The onset of rotating magneto-convection
- What tells us an experiment ?

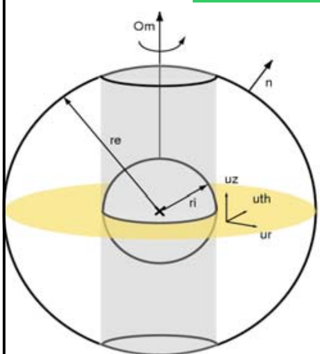
3 – <u>Impact of the magnetic field</u>
- On the vortex geometry
- On the heat flux
- On the convective motions scaling

Definition of the main parameters



Dimension-less number	Definition	Expression	Experimental values	Numerical values
Rayleigh	Buoyancy vs diffusion	$R = \frac{g\alpha\Delta T d^3}{\kappa\upsilon}$	up to 4.Rc	up to 20.Rc
Thermal Ekman	thermal diff. vs Coriolis	$E_t = \frac{\kappa}{\Omega D^2}$	down to $1.6\ 10^{-5}$	down to $1.6\ 10^{-5}$
Elsasser	Lorents vs Coriolis	$\Lambda = \frac{\sigma B^2}{\rho\Omega}$	up to $6.2\ 10^{-3}$	up to 0.1
Prandtl	viscous diff. vs thermal diff.	$P = \frac{\upsilon}{\kappa}$	0.025	7.0 0.3 0.025
Magnetic Prandtl	viscous diff. vs magnetic diff.	$P_m = \frac{\upsilon}{\lambda}$	$1.5\ 10^{-6}$	$1.5\ 10^{-6}$

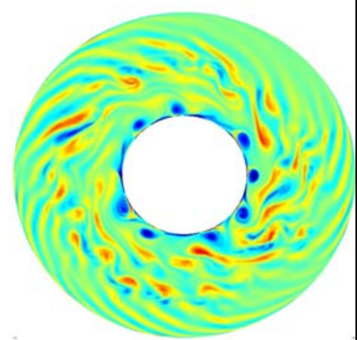
1 – Quasi-Geostrophic (QG) velocities



$$\frac{1}{P}\left(\frac{d\omega}{dt}+\frac{2\eta}{E_t}u'_s\right)=\nabla^2\omega-R\frac{\partial\Theta}{\partial\phi}$$

$$\frac{d\Theta}{dt}=\nabla^2\Theta-u'_s\frac{\partial T_s}{\partial s}$$

Exemple of **vorticity map** in the equatorial plane
P = 0.025 – R = 3.0 Rc
Et = 3.2 10^{-5}



- **Differential heating (z-integrated temperature profile)**
- **No-slip boundary conditions**
- **Velocities described in the equatorial plane**
- **Refs:** *Cardin & Olson 1994, Aubert et al 2003*
- **NB: the Ekman pumping is included**

$$T=T_s(s)+\Theta$$

$$\vec{u}=u'_s.\vec{e}_s+(u'_\phi+\bar{u}_\phi).\vec{e}_\phi$$

$$\omega=\nabla_\times\vec{u}.\vec{e}_z$$

1 – The onset of rotating convection

***Busse 1970* tells us that the critical parameters (Rayleigh number, mode and pulsation) evolve as:**

$$R_c \approx E_P^{-4/3} \quad ; \quad m_c \approx E_P^{-1/3} \quad ; \quad \omega_c \approx E_P^{-2/3}$$

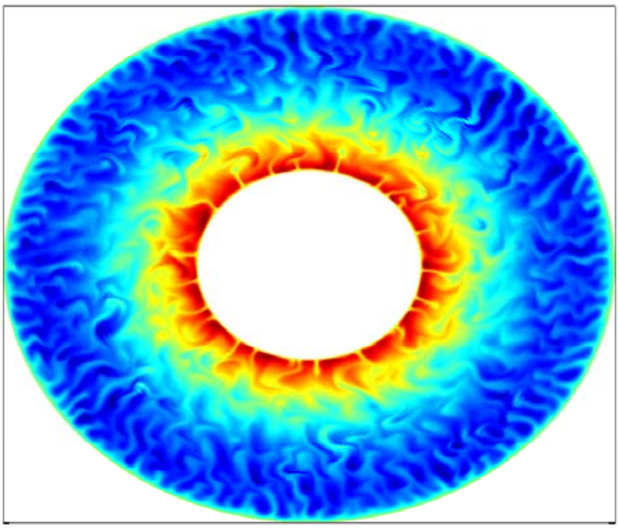
with

$$E_P = \frac{(\kappa + \upsilon)}{\eta.\Omega.d^2}$$

In the case of liquid metals, where P << 1, we can consider that

$$E_P = \frac{\kappa}{\eta.\Omega.d^2} = \frac{E_t}{\eta}$$

1 – Considerations about different sizes.
work in collaboration with Chris Jones



Temperature perturbation Θ
P = 7.0 - E = 2.6 10^{-6} - R = 17.4 Rc
m_{crit} = 22 - Nu = 10.7 - Pe = 714

- *L* the size of the convection area
- $l_{crit}=2\pi s_i / m_{crit}$ the critical size
- l_s the radial vortex size
- l_ϕ the azimutal vortex size

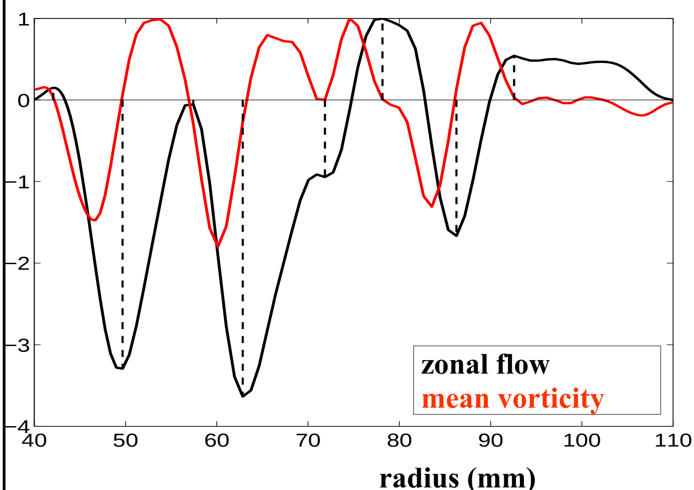
• Even far from criticallity the number of cells remains close to the critical mode :

$$l_{crit} \sim l_\phi$$

• The vortex are roughly circular :

$$l_s \sim l_\phi \sim l_{crit}$$

1 – Considerations about the vortex size.



For strong enough motions, l_s is linked to the zonal wind structure…

… thus the radial size of the zonal flow is linked to l_{crit} !!

This disagrees with the inertial scaling (*Aubert 2001*) that takes into account the Rhines scale (Reynolds stress ~ Coriolis**)…**

… the non-linearities we consider from now come from the $\vec{u}.\overrightarrow{\nabla\Theta}$ term of the heat equation

1 – Scaling of the convective motions

The mean part of the heat equation leads to define the Nusselt number as:

$$\mathrm{Nu} = \frac{\text{convective heat flux}}{\text{static heat flux}}$$

$$Nu - 1 = \frac{\nabla\overline{\Theta} + \overline{u'_s\,\Theta'}}{\nabla T_s} = \frac{\nabla\overline{\Theta}(s_i)}{\nabla T_s(s_i)}$$

$$Nu - 1 \approx L.u'_s.\Theta'$$

… from which we consider the heat-flux based Rayleigh number

$$R_Q = (Nu - 1).R$$

A Hopf bifurcation from onset leads to :

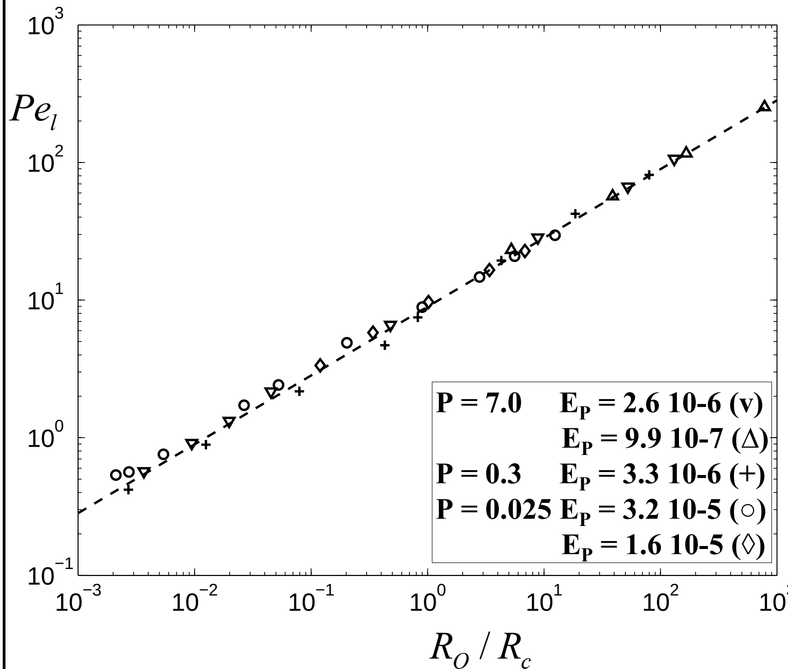
$$\mathrm{Pe} = \frac{u'_s.d}{\kappa} \approx k.\left(\frac{R_Q}{R_c}\right)^{1/2}$$

where k is the global wave number.

Since vortex are roughly circular $k \approx m_c$; then we obtain :

$$Pe_l = \frac{u'_s.l_{crit}}{\kappa} \approx \left(\frac{R_Q}{R_c}\right)^{1/2}$$

1 – Scaling of the convective motions



This scaling is verified by our QG simulations even for low values of *P* (whereas the Reynolds stress is strong)

2 – QG velocities / 3D magnetic Field

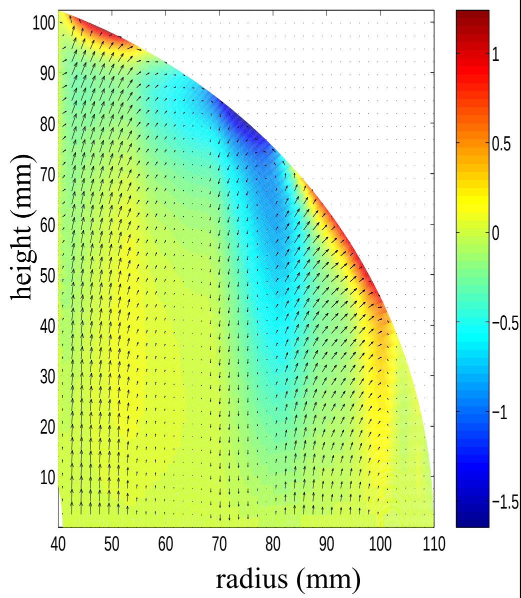
- **Imposed azimutal magnetic field $B_0 \sim 1/s$**
- **Isolating boundary conditions j.n=0**
  (main difference with *Petry et al 1997*)
- **Electrical currents j described in the whole sphere through a potential V and the Ohm's law**

$$\vec{j} = \sigma(-\overrightarrow{\nabla V} + \vec{u} \times \overrightarrow{B_0})$$

$$\nabla.\vec{j} = 0 \Rightarrow \Delta V = f(B_0, u_s)$$

- **Lorentz force jxB0 described in the vorticity equation**

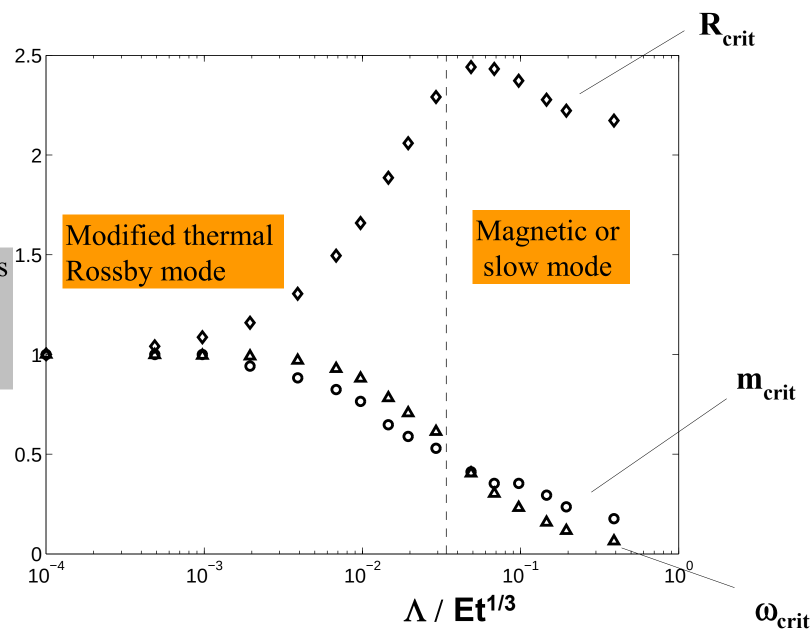
$$\frac{1}{P}\left(\frac{d\omega}{dt} + \frac{2\eta}{E_t}u'_s\right) = \nabla^2\omega - R\frac{\partial\Theta}{\partial\phi} + \frac{\Lambda}{P.E_t}\frac{B'_0}{s}\frac{\partial}{\partial\phi}\left(B'_0.u'_s + \frac{V}{L}\right)$$



Potential map (in µV) and associated electrical currents in a slice of the sphere
Pr = 0.025 – Et = 3.2 10^{-5}
L = 3.9 10^{-2} – R = 3.0 Rc

2 – The onset of rotating magneto-convection

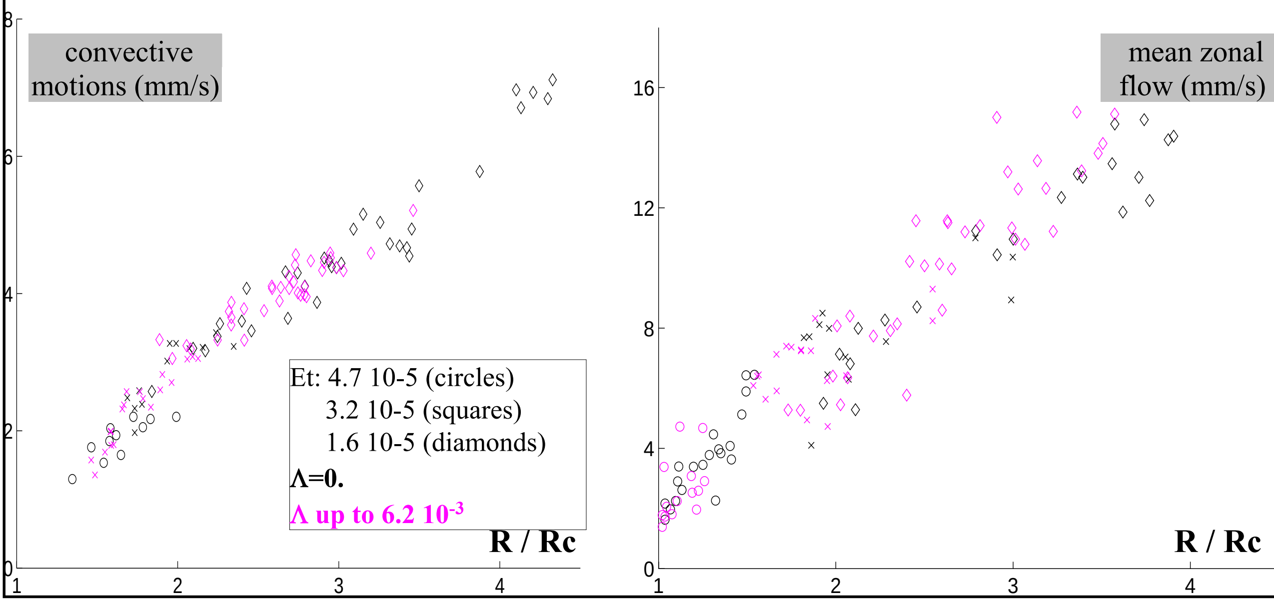
Relative critical parameters from hybrid QG / 3D calculations
Pr = 0.025 – Et = 1.6 10^{-5}



Evolution of the critical parameters in agreement with the asymptotics (*Fearn 1979, Soward 1979*)

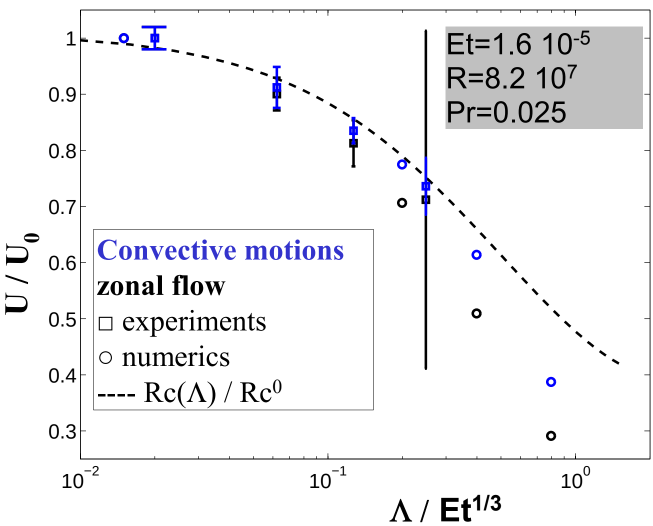
2 – What tells us our experiment ?

From Doppler ultrasonor velocimetry in the equatorial plane (*Brito 2001*)



2 – First order effect of the magnetic field

I fix R and increase Λ...



… I observe a drop of both the radial and zonal motions intensity…

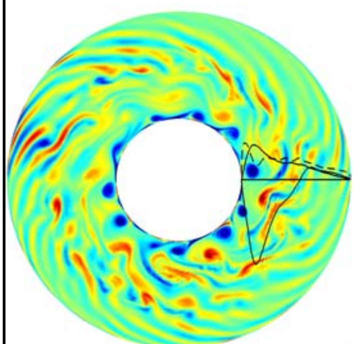
… the evolution of Rc(Λ) explains the modification of both the convective and mean zonal motions for $\Lambda < O(Et^{1/3})$!!

But a departure from this behaviour is observed in the numerical simulations as Λ reaches $O(Et^{1/3})$.

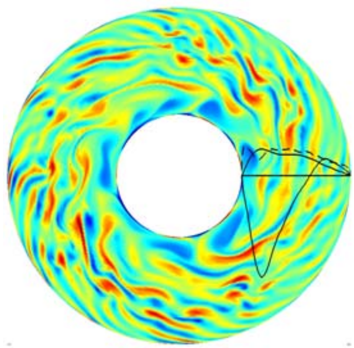
But: until now, we made no considerations about the vortex sizes and the heat flux organistation…

3 – effect of the magnetic field on the velocity field and the vortex geometry

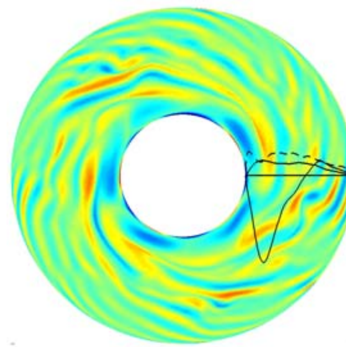
$Et = 1.6\ 10^{-5}$ – $R = 3.0\ Rc$



$\Lambda = 0.$ - $m_{crit} = 13$



$\Lambda = 3.9\ 10^{-2}$ – $m_{crit} = 8$



$\Lambda = 9.8\ 10^{-2}$ – $m_{crit} = 5$

As Λ increases:

- **u'_ϕ becomes higher than u'_s: motions prefer not to cross the magnetic field lines.**
  … as a consequence each vortex becomes more and more "beam-shaped".

- **The motions reach larger radius areas, where the magnetic field is weaker.**
  … thus the radial size *L* of the convective zone grows.

3 – effect of the magnetic field: interpretation of the heat flux evolution

- For a given R/Rc, the heat flux depends on the magnetic field.

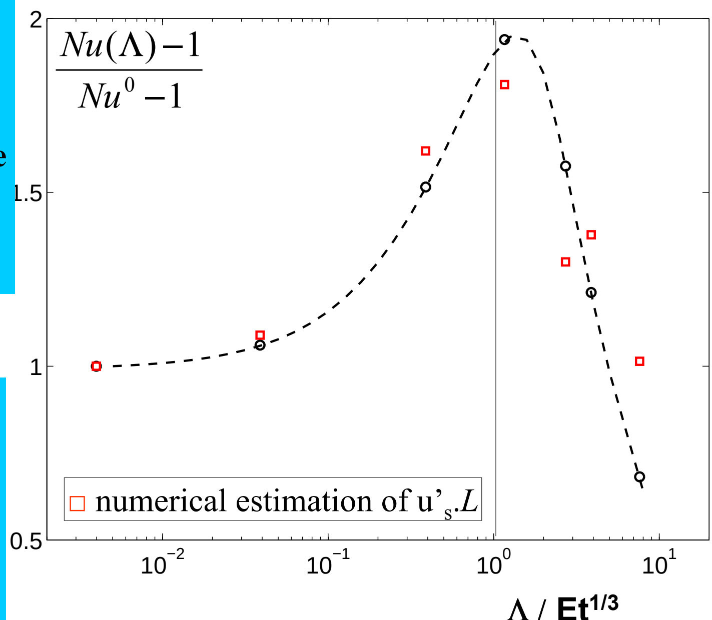
- Even in the developped convection the $Et^{1/3}$ limit seems to be the relevant parameter.

$$Nu - 1 \approx L.u'_s.\Theta'$$

This behaviour can be understood by a competition between:

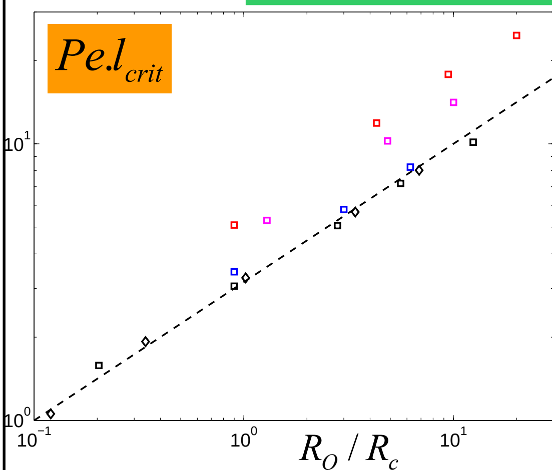
- the increase of *L* due to the magnetic field geometry ($B_0 \sim 1/s$)

- the decrease of u'_s due to the azimutal direction of B_0



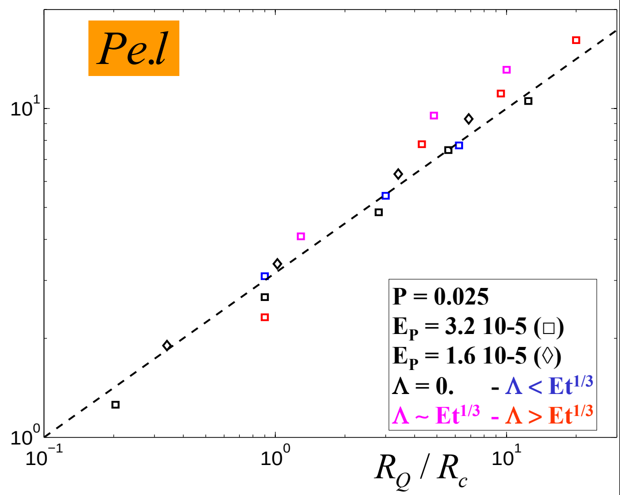
$Et = 1.610^{-5}$ - P = 0.025 - R = 5.0 Rc
from QG / 3D calculations

3 – effect of the magnetic field: come back on the velocity scaling



$$\text{with } l \approx k^{-1} \approx \left(\frac{1}{l_s^{\,2}} + \frac{1}{l_\phi^{\,2}} \right)^{1/2}$$

$$\text{we obtain still : } Pe_l = \frac{u'_s . l}{\kappa} \approx \left(\frac{R_Q}{R_c} \right)^{1/2}$$



- **As Λ increases, a systematic trend is observed if we define Pe_l from l_{crit}.**
- **Taking into account the vortex shape allows us to keep our previous physical interpretation.**