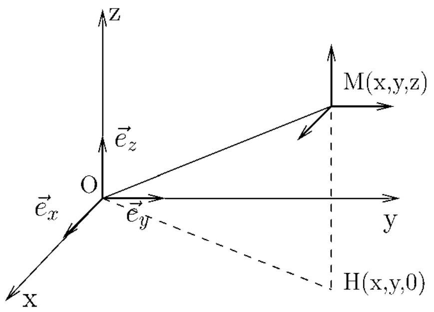


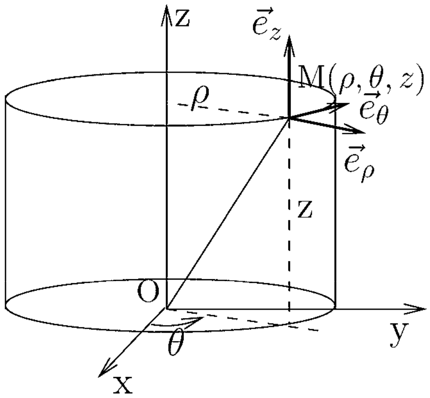


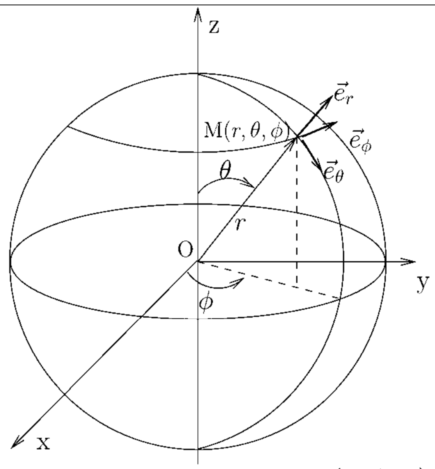


| | | |
|---|---|---|
| Coordonnées cartésiennes $(x, y, z)$ | Coordonnées cylindriques $(\rho, \theta, z)$ | Coordonnées sphériques $(r, \theta, \phi)$ |
| $\overrightarrow{OM} = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z$ <br> $d\overrightarrow{OM} = dx\vec{e}_x + dy\vec{e}_y + dz\vec{e}_z$ | $\overrightarrow{OM} = \rho\vec{e}_\rho + z\vec{e}_z$ <br> $d\overrightarrow{OM} = d\rho\vec{e}_\rho + \rho d\theta\vec{e}_\theta + dz\vec{e}_z$ | $\overrightarrow{OM} = r\vec{e}_r$ <br> $d\overrightarrow{OM} = dr\vec{e}_r + rd\theta\vec{e}_\theta + r\sin\theta d\varphi\vec{e}_\varphi$ |
| $\left(\overrightarrow{\text{grad}} f\right)_x = \frac{\partial f}{\partial x}$ <br> $\left(\overrightarrow{\text{grad}} f\right)_y = \frac{\partial f}{\partial y}$ <br> $\left(\overrightarrow{\text{grad}} f\right)_z = \frac{\partial f}{\partial z}$ | $\left(\overrightarrow{\text{grad}} f\right)_\rho = \frac{\partial f}{\partial \rho}$ <br> $\left(\overrightarrow{\text{grad}} f\right)_\theta = \frac{1}{\rho}\frac{\partial f}{\partial \theta}$ <br> $\left(\overrightarrow{\text{grad}} f\right)_z = \frac{\partial f}{\partial z}$ | $\left(\overrightarrow{\text{grad}} f\right)_r = \frac{\partial f}{\partial r}$ <br> $\left(\overrightarrow{\text{grad}} f\right)_\theta = \frac{1}{r}\frac{\partial f}{\partial \theta}$ <br> $\left(\overrightarrow{\text{grad}} f\right)_\varphi = \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}$ |
| $\text{div}\overrightarrow{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ | $\text{div}\overrightarrow{A} = \frac{1}{\rho}\left[\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{\partial A_\theta}{\partial \theta}\right] + \frac{\partial A_z}{\partial z}$ | $\text{div}\overrightarrow{A} = \frac{1}{r^3}\frac{\partial}{\partial r}\left(r^2 A_r\right)$ <br> $+\frac{1}{r\sin\theta}\frac{\partial(A_\theta\sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}$ |
| $\left(\overrightarrow{\text{rot}}\overrightarrow{A}\right)_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}$ <br> $\left(\overrightarrow{\text{rot}}\overrightarrow{A}\right)_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}$ <br> $\left(\overrightarrow{\text{rot}}\overrightarrow{A}\right)_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$ | $\left(\overrightarrow{\text{rot}}\overrightarrow{A}\right)_\rho = \frac{1}{\rho}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}$ <br> $\left(\overrightarrow{\text{rot}}\overrightarrow{A}\right)_\theta = \frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}$ <br> $\left(\overrightarrow{\text{rot}}\overrightarrow{A}\right)_z = \frac{1}{\rho}\left[\frac{\partial(\rho A_\theta)}{\partial\rho} - \frac{\partial A_\rho}{\partial\theta}\right]$ | $\left(\overrightarrow{\text{rot}}\overrightarrow{A}\right)_r = \frac{1}{r\sin\theta}\left[\frac{\partial(A_\varphi\sin\theta)}{\partial\theta} - \frac{\partial A_\theta}{\partial\varphi}\right]$ <br> $\left(\overrightarrow{\text{rot}}\overrightarrow{A}\right)_\theta = \frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{1}{r}\frac{\partial(rA_\varphi)}{\partial r}$ <br> $\left(\overrightarrow{\text{rot}}\overrightarrow{A}\right)_\varphi = \frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]$ |
| $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ | $\Delta f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) +$ <br> $\frac{1}{\rho^2}\frac{\partial^2 f}{\partial\theta^2} + \frac{\partial^2 f}{\partial z^2}$ | $\Delta f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) +$ <br> $\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}$ |

DEFINITIONS INTRINSEQUES ET EQUATIONS INTEGRALES CORRESPONDANTES

$$df = \overrightarrow{grad}\, f \cdot d\overrightarrow{OM}$$

différentielle du scalaire f

$$f_B - f_A = \int_A^B \overrightarrow{grad}\, f \cdot d\overrightarrow{OM}$$

$$d\Phi_{\overrightarrow{A}} = div\ \overrightarrow{A}\ d\tau$$

(différentielle du flux du vecteur $\overrightarrow{A}$
à travers une surface fermée)

$$\oint_S \overrightarrow{A} \cdot d\overrightarrow{S} = \int_V div\ \overrightarrow{A}\ d\tau$$

$$dC_{\overrightarrow{A}} = \overrightarrow{rot}\,\overrightarrow{A} \cdot d\overrightarrow{S}$$

(différentielle de la circulation
du vecteur $\overrightarrow{A}$ sur un contour fermé)

$$\oint_C \overrightarrow{A} \cdot d\overrightarrow{OM} = \int_S \overrightarrow{rot}\,\overrightarrow{A} \cdot d\overrightarrow{S}$$

4 IDENTITES

$$\overrightarrow{rot}\left(\overrightarrow{grad}\, f\right) \equiv \overrightarrow{0}$$

$$div\left(\overrightarrow{rot}\ \overrightarrow{A}\right) \equiv 0$$

$$div\left(\overrightarrow{grad}\, f\right) \equiv \Delta f$$

$$\overrightarrow{rot}\left(\overrightarrow{rot}\ \overrightarrow{A}\right) \equiv \overrightarrow{grad}\left(div\ \overrightarrow{A}\right) - \overrightarrow{\Delta}\,\overrightarrow{A}$$

Quelques formules utiles

$$\overrightarrow{grad}\ (f\,g) = \left(\overrightarrow{grad}\, f\right) g + f\,\overrightarrow{grad}\, g$$

$$div\left(f\ \overrightarrow{A}\right) = \left(\overrightarrow{grad}\, f\right) \cdot \overrightarrow{A} + f\, div\ \overrightarrow{A}$$

$$\overrightarrow{rot}\left(f\ \overrightarrow{A}\right) = \left(\overrightarrow{grad}\, f\right) \wedge \overrightarrow{A} + f\,\overrightarrow{rot}\ \overrightarrow{A}$$

$$div\left(\overrightarrow{A} \wedge \overrightarrow{B}\right) = \left(\overrightarrow{rot}\ \overrightarrow{A}\right) \cdot \overrightarrow{B} - \overrightarrow{A} \cdot \overrightarrow{rot}\ \overrightarrow{B}$$